Dropping the lowest grade for the homework won't decrease the homework average score

To prove this, assume that there are n homeworks assigned and that $hw_1, hw_2, \ldots, hw_n$ are the scores for these assignments. Without loss of generality we can assume that

$$hw_1 \leq hw_2 \leq \cdots \leq hw_n. \tag{1}$$

The average homework score is $\frac{hw_1+hw_2+\cdots+hw_n}{n}$. The average homework score after dropping the smallest score (this score is hw_1 under our assumptions) is $\frac{hw_2+hw_3\cdots+hw_n}{n-1}$.

The above statement will follow from the next theorem.

Theorem 1

$$\frac{hw_1 + hw_2 + \cdots + hw_n}{n} \leq \frac{hw_2 + hw_3 \cdots + hw_n}{n-1}. \tag{2}$$

Proof:

Multiplying the left-hand side of (2) by $n-1$ and the right-hand side by n, we get that the inequality (2) is equivalent to

$$hw_1 \cdot (n-1) + hw_2 \cdot (n-1) + \cdots + hw_n \cdot (n-1) \leq hw_2 \cdot n + hw_3 \cdot n + \cdots + hw_n \cdot n. \tag{3}$$

Moving the terms $hw_2 \cdot (n-1) + \cdots + hw_n \cdot (n-1)$ to the right-hand side of (3) results in

$$hw_1 \cdot (n-1) \leq hw_2 \cdot n + hw_3 \cdot n + \cdots + hw_n \cdot n - (hw_2 \cdot (n-1) + \cdots + hw_n \cdot (n-1)),$$

or, after grouping the terms,

$$hw_1 \cdot (n-1) \leq hw_2 \cdot (n-(n-1)) + hw_3 \cdot (n-(n-1)) + \cdots + hw_n \cdot (n-(n-1)). \tag{4}$$

Simplifying (4) we get that to prove (2) is equivalent to prove

$$hw_1 \cdot (n-1) \leq hw_2 + hw_3 + \cdots + hw_n. \tag{5}$$

Now, (5) can be rewritten as

$$\underbrace{hw_1 + hw_1 + \cdots + hw_1}_{n-1 \text{ times}} \leq hw_2 + hw_3 + \cdots + hw_n. \tag{6}$$

Moving all terms from the left-hand side of (6) to the right-hand side, we get that (6) is equivalent to

$$0 \leq (hw_2 - hw_1) + (hw_3 - hw_1) + \cdots + (hw_n - hw_1). \tag{7}$$

Remember (1) that hw_1 is the minimum score, so $hw_i - hw_1 \geq 0$ for $i = 2, 3, \ldots, n$. Therefore, every term in (7) is non-negative. Hence, the entire sum of non-negative terms is also non-negative, and (7) is established.

Since (7) is equivalent to (2) the Theorem is proved. $\square$