#### **Reducing Problems**

We have seen that polynomial time reduction between problems is a very useful concept for studying relative complexity of problems. It allowed us to distinguish a class of problems, **NP**, which includes many important problems and is viewed as the class of hard problems

We are going to do the same for space complexity classes: **NL** and **PSPACE** 

There is a problem:

Polynomial time reduction is too powerful

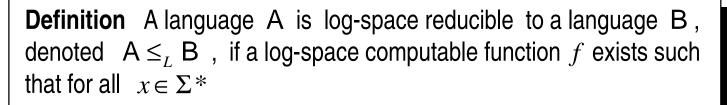
## **Log-Space Reduction**

A transducer is a 3-tape Turing Machine such that

- the first tape is an input tape, it is never overwritten
- the second tape is a working tape
- the third tape is an output tape, no instruction of the transition function uses the content of this tape

The space complexity of such a machine is the number of cells on the working tape visited during a computation

A function  $f: \Sigma^* \to \Sigma^*$  is said to be log-space computable if there is a transducer computing f in  $O(\log n)$ 



$$x \in \mathsf{A} \Leftrightarrow f(x) \in \mathsf{B}$$

Note that a function computable in log-space is computable in polynomial time, so

$$\mathsf{A} \leq_L \mathsf{B} \Longrightarrow \mathsf{A} \leq \mathsf{B}$$

## Completeness

#### Definition

A language L is said to be NL-complete if L  $\in$  NL and, for any A  $\in$  NL,

$$\mathsf{A} \leq_L \mathsf{L}$$

# DefinitionA language L is said to be P-complete if $L \in P$ and, for any<br/> $A \in P$ , $A \in P$ , $A \leq_L L$